國立中山大學九十一學年度博士班招生考試試題

科目：量子力學【物理系】

Some might-be-useful informations:

1. for simple harmonic oscillator:

$$\psi_E(x) \equiv \psi_{(n+1/2)\hbar\omega}(x) \equiv \psi_n(x)$$
$$= \left(\frac{m\omega}{\pi\hbar 2^{2n}(n!)^2}\right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right) H_n\left[\left(\frac{m\omega}{\hbar}\right)^{1/2} x\right]$$

$$\int_{-\infty}^{\infty} H_n(y) H_{n'}(y)\, e^{-y^2}\, dy = \delta_{nn'}(\pi^{1/2} 2^n n!)$$

$$a = \left(\frac{m\omega}{2\hbar}\right)^{1/2} X + i\left(\frac{1}{2m\omega\hbar}\right)^{1/2} P$$

$$a^\dagger = \left(\frac{m\omega}{2\hbar}\right)^{1/2} X - i\left(\frac{1}{2m\omega\hbar}\right)^{1/2} P$$

2. for hydrogen atom:

$$\psi_{1,0,0} = \left(\frac{1}{\pi a_0^3}\right)^{1/2} e^{-r/a_0}$$
$$\psi_{2,0,0} = \left(\frac{1}{32\pi a_0^3}\right)^{1/2} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$$
$$\psi_{2,1,0} = \left(\frac{1}{32\pi a_0^3}\right)^{1/2} \frac{r}{a_0} e^{-r/2a_0} \cos\theta$$
$$\psi_{2,1,\pm 1} = \mp\left(\frac{1}{64\pi a_0^3}\right)^{1/2} \frac{r}{a_0} e^{-r/2a_0} \sin\theta\, e^{\pm i\phi}$$

Each problem weights 20 points.

1. Find the transmission coefficient for a rectangular potential barrier (Fig. 1.) if the

energy exceeds the height V of the barrier. *Fig. 1*

2. Using perturbation theory to find the eigenstate's correction up to first order and the eigen energy's correction up to second order for a one –dimensional simple harmonic oscillator under a small constant force f.

3. A particle of spin 1 and a particle of spin 2 are at rest in a configuration such that the total spin is 3, and its z-component is 1 (we take $\hbar = 1$). If you measured the z-component of the angular momentum of the spin-2 particle, what values might you got on what probabilities.

4. A hydrogen atom in its ground state is placed in a homogeneous electric field that has the time dependence: $\vec{E} = 0, \quad t < 0$

$$\vec{E} = \vec{A}\exp(-t/b), \quad t > 0.$$

Find the first-order probability of the atom being at 2p states after a long time.

5. Prove a Hermitian operator has only real eigen values and its eigenstates are orthogonal in a non-degenerate case.

國立中山大學九十一學年度博士班招生考試試題

科目：電動力學【物理系】

1. A planar charge density is given by $\sigma(z)=\rho_0 z\exp(-\eta|z|)$, obtain the electrostatic potential, $\Phi(z)$, with the boundary condition $\Phi(z)=0$ when $z\rightarrow\infty$. ρ_0 and η are positive constants. 20%
2. If the electronic charge density of an atom with an atomic number of Z is given by $\rho_e(r,\theta,\phi)=(\rho_0+\rho_1\sin\theta\cos\phi)\exp\left(-r/a_0\right)$, where $\rho_0 = Ze/(8\pi a_0{}^3)$ and ρ_1 is a small constant. Obtain the electrostatic potential, $\Phi(r,\theta,\phi)$, in terms of spherical harmonics. [Hint: $\frac{1}{|\vec{r}-\vec{r}'|}=4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}^*(\theta',\phi')Y_{lm}(\theta,\phi)$; $Y_{00}(\theta,\phi)=\frac{1}{\sqrt{4\pi}}$, $Y_{11}(\theta,\phi)=-\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\phi}$, $Y_{10}(\theta,\phi)=\sqrt{\frac{3}{4\pi}}\cos\theta$, and $Y_{1-1}(\theta,\phi)=\sqrt{\frac{3}{8\pi}}\sin\theta e^{-i\phi}$] 20%
3. A point charge q is placed a distance d away from the surface of a semi-infinite dielectric material with a dielectric constant of ε. Using the image charge method to find the electrostatic potential at all points in space. 20%
4. Using the symmetry and the Ampère's law to find the magnetic induction, **B**(**r**), given rise by a current density $\mathbf{J}(\rho)=J_0\mathbf{k}\exp(-\eta\rho)$ in cylindrical coordinates (ρ,ϕ,z), where **k** is a unit vector in the z direction and J_0 and η are positive constants. 20%
5. Derive the boundary conditions of the magnetic induction, **B**, and the magnetic field, **H**, at the interface of two mediums with magnetic permeability of μ_1 and μ_2, respectively. 5%
6. Using Maxwell equations to derive the wave equations for the scalar potential, $\Phi(\mathbf{r})$, and the vector potential, $\mathbf{A}(\mathbf{r})$, in free space. 15%