**20% for each problem**

1. (a). Show that, for any two operators A and L,

$$e^{L}Ae^{-L} = A + [L, A] + \frac{1}{2!}[L,[L, A]] + \frac{1}{3!}[L,[L,[L, A]]] + \ldots$$

(b). Treating the coordinate x as an operator in the Schrodinger picture, determine the corresponding operator $x_H$ in the Heisenberg picture (i) for the free particle and (ii) for the harmonic oscillator.

2. Typically, the interaction potential depends only on the vector $\vec{r} \equiv \vec{r_1} - \vec{r_2}$ separating the two particles. In that case the Schrodinger equation separates, if we change variables from $\vec{r_1}, \vec{r_2}$ to $\vec{r}, \vec{R}$, where $\vec{R}$ is the center of mass.

(a) Show that $\vec{r_1} = \vec{R} + (\frac{\mu}{m_1})\vec{r_1}$, $\vec{r_2} = \vec{R} + (\frac{\mu}{m_2})\vec{r_2}$ and $\nabla_1 = (\frac{\mu}{m_2})\nabla_R + \nabla_r$, $\nabla_2 = (\frac{\mu}{m_1})\nabla_R + \nabla_r$, where $\mu$ is the reduced mass of the system.

(b) Show that the Schrodinger equation becomes

$$\frac{-\hbar^2}{2(m_1 + m_2)}\nabla_R^2\psi - \frac{\hbar^2}{2\mu}\nabla_r^2\psi + V(\vec{r})\psi = E\psi$$

( c) Solve by separation of variables, letting $\psi(\vec{R}, \vec{r}) = \psi_R(\vec{R})\psi_r(\vec{r})$, and explain the meaning of your results.

3. Solve the infinite cubical well (or "particle in a box"):

$V(x,y,z) = \begin{cases} 0, \\ \infty \end{cases}$ if x, y, z are all between 0 and a, the V is 0. Otherwise the V is infinite.

(a) Find the stationary state wave functions and the corresponding energies.

(b) Call the distinct energies $E_1, E_2, E_3, \ldots$, in order of increasing energy. Find $E_1, E_2, E_3, E_4, E_5, E_6$. Determine the degeneracy of each of these energies.

4. The energy representation of Hamiltonian matrix is as follows: $\begin{pmatrix} E_1 & b \\ b & E_2 + a \end{pmatrix}$

   (a) Find the energy with the perturbation method to the second order.

   (b) Solve energy in exact method and compare with the result of (a).

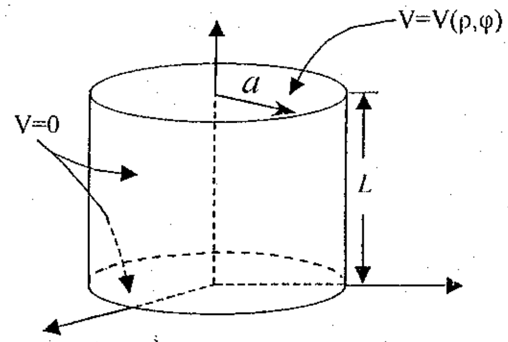
5. (a). Show that $[L_x, L_y] = i\hbar L_z$, for angular momenta.

   (b) An electron is in spin state $\chi = A\begin{pmatrix} 3i \\ 4 \end{pmatrix}$. Find the normalization constant A.

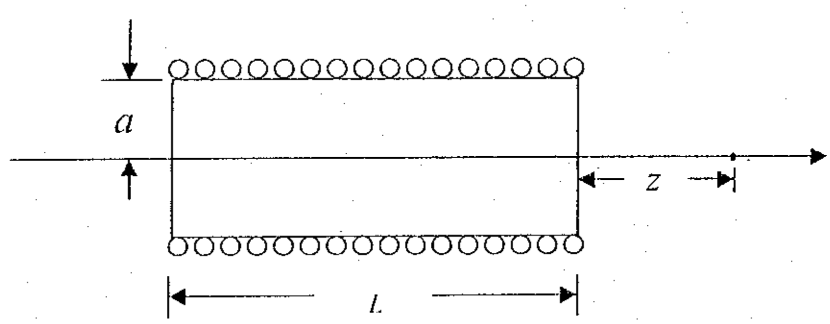
# 國立中山大學九十學年度博士班招生考試試題

科目：電動力學【物理所】 

1. A Cylindrical cavity with a radius of $a$ and height of $L$ is positioned at the origin as shown in the figure 1. The top surface is isolated to the side surface of the cylinder. The charge distribution of the top surface is charged in such a way that its electric potential depends on the radius ρ and angle φ. The other two surfaces, the side and bottom surfaces, are kept at zero potential. Please find the electric potential (V) and field (**E**) inside and outside of the cylinder. ( 30% )




2. (a) A cylindrical coil is formed by N turns of single coil of radius of $a$ in a total length of $L$. If a current $I$ is applied in the coil. Please find the magnetic flux density (**B**) and the magnetic vector potential (**A**) at the point located at the cylindrical axis and z distance to the edge of the coil. (15% )
   (b) What will be the results if the cylindrical coil is replaced by a cylinder magnet with a uniform magnetization **M** along its axis. ( 15% )



3. (a) Please write down complete forms and names of all Maxwell Equations. (10%)
   (b) What will the modified Maxwell equation look like if the magnetic monopole does exist? (10%)
   (c) An oblique incident of polarized EM wave from material 1, (μ,ε), into material 2, (μ',ε'). The *E* of the polarized wave in parallel to the plane of incident. Please prove that (20%)

$$\frac{E_{transmission}}{E_1}=\frac{2nn'\cos(i)}{\frac{\mu}{\mu'}n'^2\cos(i)+n\sqrt{n'^2-n^2\sin^2(i)}}$$

$$\frac{E_{reflection}}{E_1}=\frac{\frac{\mu}{\mu'}n'^2\cos(i)-n\sqrt{n'^2-n^2\sin^2(i)}}{\frac{\mu}{\mu'}n'^2\cos(i)+n\sqrt{n'^2-n^2\sin^2(i)}}$$

Bessel Function

§ $x^2 y'' + x y' + (x^2 - \nu^2) y = 0$ $\nu \in$ real and +

令 $y(x) = \sum_{m=0}^{\infty} a_m x^{m+r}$ $a_0 \neq 0$

§ $y(x) = J_\nu(x) = x^\nu \sum_{s=0}^{\infty} \frac{(-1)^s x^{2s}}{2^{2s+\nu} s! \, \Gamma(\nu+s+1)}$ Bessel Func. of 1st kind of order $\nu$

§ If $\nu \neq$ Integral

$y(x) = C_1 J_\nu(x) + C_2 J_{-\nu}(x)$

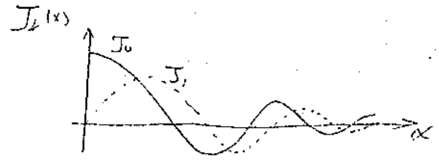
If $\nu =$ Integral

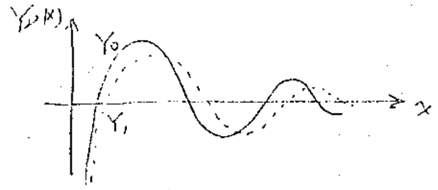
$y(x) = C_1 J_\nu(x) + C_2 Y_\nu(x)$

$\because J_{-\nu}(x) = (-1)^\nu J_\nu(x)$

and $Y_\nu(x) = \frac{1}{\sin \nu\pi} \left[ J_\nu(x) \cos \nu\pi - J_{-\nu}(x) \right]$

Bessel Func. of 2nd kind of order $\nu$




§ $\int_0^a \rho \, J_m\left(x_{mn} \frac{\rho}{a}\right) J_m\left(x_{mn'} \frac{\rho}{a}\right) d\rho = \frac{a^2}{2} \left[ J_{m+1}(x_{mn})^2 \right] \delta_{nn'}$