國立中山大學八十七學年度碩博士班招生考試試題

科　目：量子力學

物理學系　　共 1 頁 第 1 頁

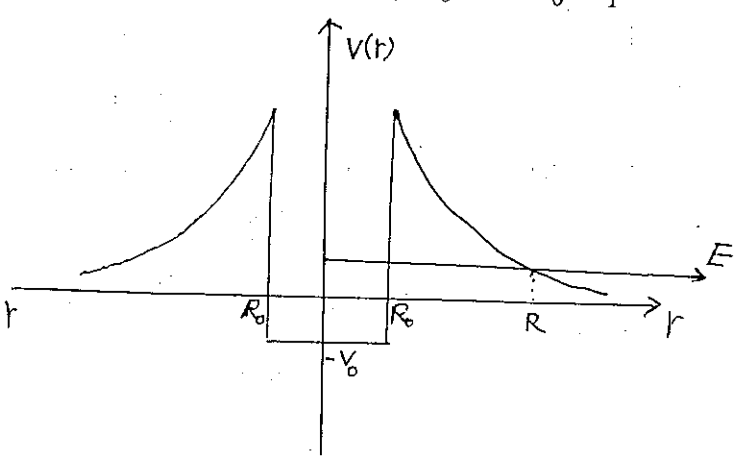
1. An incident particle of charge q_1 with a kinetic energy of E scatters off a heavy stationary particle of charge q_2. Use the Born approximation to determine the differential scattering cross-section. (20%)

2. Use the variational principle and gaussian trial wave function $\psi(x) = Ae^{-bx^2}$ to find the lowest bound of the ground state of (a) a one- dimensional harmonic oscillator and (b) a particle in the delta-function potential $V(r) = -\alpha\delta(x)$ (20%)

3. The fact that α-particles having energies of a few MeV can leave potential wells with depths of tens of MeV (inside which they find themselves in radioactive nuclei), can be explained by the tunnel effect. Using a simplified potential $V(r) = -V_0, (r < R_0); V(r) = \frac{e_1 e_2}{r}, (r > R_0)$ (Fig.1), calculate the transmission probability for α-particles of energy E through the barrier and estimate the mean life of an α-emitting nucleus. (20%)

4. At $t < 0$, an electron is known to be in the $n = 1$ eigenstate of a one dimensional infinite square well potential, which extends from $x = 0$ to $x = a$. At $t = 0$, a uniform electric field E is applied in the direction of increasing x. The electric field is left on for a short time τ and then removed. Use time dependent perturbation theory to calculate the probability that the electron will be in the $n = 2, 3,$ or 4 eigenstates at $t > \tau$. (20%)

5. Suppose a spin 1/2 particle is in the state $\chi = \frac{1}{\sqrt{6}}\begin{pmatrix} 1+i \\ 2 \end{pmatrix}, i = \sqrt{-1}$, find the expectation values of the three components of spin **S** (i.e. $\langle S_x\rangle, \langle S_y\rangle, and \langle S_z\rangle$). (note Pauli spin matrices is $\sigma_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$) (20%)



Note. (1) $\int_{-\infty}^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$　　(2) $\int_{-\infty}^{\infty} e^{-x^2}x^2dx = \frac{\sqrt{\pi}}{4}$

1. A homogeneous sphere of radius a and permittivity ε_1 is placed in a homogeneous dielectric medium of permittivity ε_2. At a large distance from the sphere, the electric field E_0 in the dielectric is uniform. (a) Find the potential everywhere (inside and outside the sphere). (8 points) (b) Find the electric field everywhere. (8 points) (c) Find the distribution of bound charges. (5 points)
2. Derive the wave equation from the Maxwell equations. (10 points)
3. (a) A charge particle of mass m and charge q is moving in an electromagnetic field (**E** and **B**). Write out its equation of motion. (5 points) (b) When the field is only magnetic, show that the motion is periodic and obtain its frequency. (10 points)
4. Consider a conductor with conductivity σ, dielectric constant ε_0, and magnetic permeability μ, (a) for a "good" conductor, calculate the phase difference between the magnetic field and the electric field of an electromagnetic wave. (b) Determine the ratio of their amplitudes. (c) What is the skin depth of the good conductor? (10 points)
5. The **E** field in a source–free region is given as $\mathbf{E} = \hat{j}E_0 \sin \alpha x \cos(\omega t - kz)$ V/m. Determine (a) the magnetic field intensity, (b) the necessary condition for the fields to exist, and (c) the time average power flow per unit area. (12 points)
6. The magnetic field intensity in free space is given as $\mathrm{H} = \hat{j}H_0 \sin(\omega t - \beta z)$ A/m, where β is a constant quantity. Determine (a) the displacement current density and (b) the electric field intensity. (8 points)
7. Find (a) the vector potential and (b) magnetic flux density at a distant point of a small circular loop of radius a that carriers current I. (16%)
8. Determine the electric field caused by a spherical cloud of electrons with a volume charge density $\rho = \rho_0$ for $0 \le R \le b$ (both ρ_0 and b are positive) and ρ=0 for R>b. (8%)