科目：**工程數學【機電系】**

工程數學部份 (100 %) （單選題：每題 5%）

1. 試問能滿足以下初始值問題 $y'(x)=e^{-x}; y(0)=2$ 之解為 （A） $y=3-e^{-2x}$ （B） $y=3-e^{-x}$ （C） $y=2-e^{-3x}$ （D） $y=2-\sin(-x)$ （E） 以上皆非

2. 試問能滿足以下初值問題 $y'(x)=y^2e^{-x}; y(1)=4$ 之解為 （A） $y=\dfrac{4}{1+4e^{-x}-4e^{-1}}$ （B） $y=3e^{-2x}+e^{-x}$ （C） $y=5x-e^{x-1}$ （D） $y=4x^2-x\sin(1-x)$ （E） 以上皆非

3. 試問能滿足以下邊界問題 $\ln(y^x)y'=3x^2y;\quad y(2)=e^3$ 之解為 （A） $y=e^{2x-1}$

 （B） $(\ln(y))^2=3x^2-3$ （C） $y=(2x-3)e^{3x-3}$ （D） $y=(4x-7)e^3$ （E） 以上皆非

4. 試問能滿足以下初值問題 $\ddot{y}+3\dot{y}=0;\quad y(0)=3, \dot{y}(0)=6$ 之解為 （A） $y=3e^{2t}$

 （B） $y=2e^{2t}+2$ （C） $y=5-2e^{-3t}$ （D） $y=3e^{-2t(1-t)}$ （E） 以上皆非

5. 以下公式中何式被稱為尤拉公式(Euler's formula) （A） $e^x=\sum_{n=0}^{\infty}\dfrac{1}{n!}x^n$

 （B） $\cos(x)=\sum_{n=0}^{\infty}\dfrac{(-1)^n}{(2n)!}$ （C） $e^{ix}=\cos(x)+i\sin(x)$ （D） $y''+p(x)y'+q(x)y=0$

 （E） 以上皆非

6. Determine the value of A so that the following equation is "exact"

$$Ay^2+ye^{xy}+(4xy+xe^{xy}+2y)y'=0$$

 (A) A=2 （B）A=5 （C）A=11 （D）A=18 （E）None

7. Which of following differential equations is a nonlinear equation

 （A） $y'+x^2y=3x$ （B） $y''+xy'+3x^2y=x$ （C） $y'+\dfrac{\sin x}{y}=2$ （D） $y''+\sin x=y$

 （E）None

8. Which of following differential equations is a linear equation

 （A） $y''+4xy'=\cos y$ （B） $y'+x^2y\sin x=3x$ （C） $y''+\sin(y)y'+y^2=2$

 （D） $yy''+xy'=3y$ （E）None

9. The Laplace Transform of the function $y(t)=4t\sin 2t$ is (A) $Y(s)=\dfrac{16s}{\left(s^2+4\right)^2}$

(B) $Y(s)=\dfrac{2}{(s+2)^2}$ (C) $Y(s)=\dfrac{s^2-2}{s^2\left(s^2+2\right)-4s}$ (D) $Y(s)=\dfrac{s}{2s^2-4}$ (E) None

10. The inverse Laplace Transform of the function $Y(s)=\dfrac{5}{(s+7)^2}$ is (A) $y(t)=5\cos(7t)$

(B) $y(t)=5te^{-7t}$ (C) $y(t)=5\sin(7t)$ (D) $y(t)=5/\sin(7t)$ (E) None

11. 有一初值問題 $y''+y=t; y(0)=1, y'(0)=0$，其對應之拉卜拉斯(Laplace Transform)式，

應為 (A) $Y(s)=\dfrac{s}{s^2+1}$ (B) $Y(s)=\dfrac{1}{s^2\left(s^2+1\right)}$ (C) $Y(s)=\dfrac{1}{s^2\left(s^2+1\right)}+\dfrac{s}{s^2+1}$

(D) $Y(s)=\dfrac{1}{s^2\left(s^2+1\right)}-\dfrac{s}{\left(s^2-1\right)}$ (E) 以上皆非

12. 試問θ角在那一象限將使$\dfrac{4+4i}{\cos\theta+i\sin\theta}$之值為一大於 0 之實數 (A) 第一象限 (B) 第二象限 (C) 第三象限 (D) 第四象限 (E) 以上皆非

13. 若令兩向量分別為$\vec{F}=a_1\vec{i}+b_1\vec{j}+c_1\underline{k}$ 與$\vec{G}=a_2\vec{i}+b_2\vec{j}+c_2\vec{k}$，則其點積(dot product)值 $\vec{F}\cdot\vec{G}$ 將為 (A) $a_1a_2+b_1b_2+c_1c_2$ (B) $\sqrt{a_1a_2+b_1b_2+c_1c_2}$ (C) $\dfrac{1}{a_1a_2+b_1b_2+c_1c_2}$ (D) $\left(a_1a_2+b_1b_2+c_1c_2\right)^2$ (E) 以上皆非

14. 若令兩向量分別為$\vec{F}=\vec{i}+2\vec{j}-3\vec{k}$ 與$\vec{G}=-2\vec{i}+\vec{j}+4\vec{k}$，則其叉積(cross product)值$\vec{F}\times\vec{G}$ 將為 (A) $2\vec{i}+11\vec{j}+5\vec{k}$ (B) $5\vec{i}+11\vec{j}+2\vec{k}$ (C) $2\vec{i}+5\vec{j}+11\vec{k}$ (D) $11\vec{i}+2\vec{j}+5\vec{k}$ (E) 以上皆非

15. 若令三個向量分別為$\vec{F}=\vec{i}-\vec{j}-\vec{k}$，$\vec{G}=-3\vec{i}+4\vec{j}+6\vec{k}$與$\vec{H}=-2\vec{i}-4\vec{j}+2\vec{k}$，則其乘積 $\vec{H}\cdot\left(\vec{F}\times\vec{G}\right)$ 將為 (A) 2 (B) 5 (C) 11 (D) 18 (E) 以上皆非

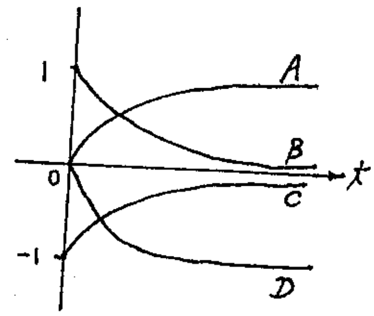
國立中山大學九十二學年度博士班招生考試試題

科目：**工程數學【機電系】**

16. 若 $f(t)=1-M^{-kt}$ 且 $M>0, k>0$，試問右圖中拿一條曲線可能代表上述方程式？

(A) 曲線 A (B) 曲線 B (C) 曲線 C

(D) 曲線 D (E) 以上皆非



17. 若兩矩陣分別為 $A=\begin{bmatrix}1 & 1 & 2 & 1\\ 4 & 1 & 6 & 2\end{bmatrix}$ 及 $B=\begin{bmatrix}-1 & 8\\ 2 & 1\\ 1 & 1\\ 12 & 6\end{bmatrix}$，試問 $(AB)^T$ 之結果為

(A) $\begin{bmatrix}15 & 28\\ 17 & 51\end{bmatrix}$ (B) $\begin{bmatrix}15 & 17\\ 28 & 51\end{bmatrix}$ (C) $\begin{bmatrix}51 & 17\\ 28 & 15\end{bmatrix}$ (D) $\begin{bmatrix}28 & 51\\ 15 & 17\end{bmatrix}$ (E) 以上皆非

18. 若一向量為 $\vec{F}=y\vec{i}+2xz\vec{j}+ze^x\vec{k}$，試問其旋度(curl) $\nabla\times\vec{F}$ 將為

(A) e^x (B) 0 (C) $2x\vec{i}-ze^x\vec{j}+2\vec{k}$ (D) $-2x\vec{i}-ze^x\vec{j}+(2z-1)\vec{k}$ (E) 以上皆非

19. 試問以下奇函數在區間上的傅立葉(Fourier)級數其前兩項為：

$$f(x)=\begin{cases}-4, & -\pi\le x\le 0\\ 4, & 0<x<\pi\end{cases}$$

(A) $\left(\frac{16}{\pi}\right)\left(\cos x+\frac{1}{3}\cos 3x+\cdots\right)$ (B) 4 (C) $\left(\frac{16}{\pi}\right)\left(\sin x+\frac{1}{3}\sin 3x+\cdots\right)$

(D) $\left(\frac{16}{\pi}\right)\left(\sin x+\frac{1}{2}\sin 2x+\cdots\right)$ (E) 以上皆非

20. 試針對一圍住 i 而不通過 i 的 任一閉路徑 Γ，以下柯西 (Cauchy) 積分值為

$$\int_\Gamma \frac{e^{z^2}}{z-i}dz$$

(A) $2\pi e^{2\pi}-1$ (B) 0 (C) 2π (D) $2\pi e^{-1}$ (E) 以上皆非

國立中山大學九十二學年度博士班招生考試試題

科目：熱傳學及流體力學【機電系甲組】

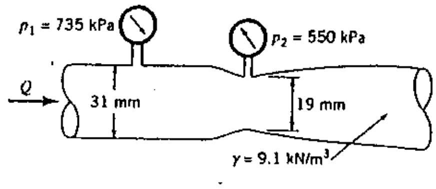
(10%) 1.

$$\bar{V} = \frac{V_0}{\ell}(x\bar{i} - y\bar{j})$$

(a) Determine the streamlines for the two-dimensional steady flow

(b) Determine the acceleration field for this flow

(10%) 2. Determine the flowrate through the Venturi meter shown in the figure if ideal conditions exist.

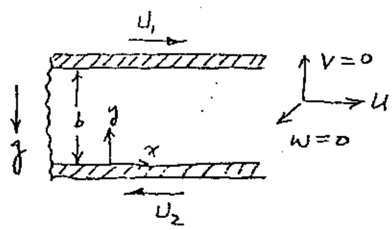


(20%) 3. An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as is shown in the figure. The two plate move in opposite directions with constant velocities, U_1 and U_2, as shown. The pressure gradient in the x-direction is zero, and the only body force is due to fluid weight. Assume laminar flow. Use the Navier-Stokes equations to derive an expression for the velocity distribution between the plates.

(a) Write down the governing equation,

(b) What are the boundary conditions for the equation in (a)

(c) Solve for the velocity distribution

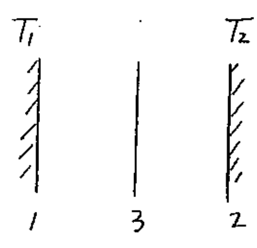


(10%) 4. A thin rectangular plate having a width w and a height h is located so that it is normal to a moving stream of fluid. Assume the drag D, that the fluid exerts on the plate is a function of w and h, the fluid viscosity and density, μ and ρ, respectively, and the velocity V of the fluid approaching the plate. Determine a suitable set of pi terms to study this problem experimentally.

(10%) 5

(a) Consider two large, black parallel walls, 1 and 2, separated by one thin black plate as show below at steady state, find the radiation energy exchange per unit area, $\dot{Q}_{12}/A = ?$

(b) Repeat the problem if the walls and the plates are gray surfaces. Use $\varepsilon_1, \varepsilon_2, \varepsilon_3$ as the emittances of the wall 1, wall 2 and the plate , respectively.



(20%) 6. An experimental device that produces excess heat is passively cooled. The addition of pin fins to the casing of this device is being considered augment the rate of cooling. Consider a copper pin fin 0.25 cm in diameter that protrudes from a wall at 95 ℃ into ambient air at 25 ℃ as shown in Fig. The heat transfer is mainly by natural convection with a coefficient equal to $10 W/m^2K$; $k = 396 W/mK$. Assume the fin is "infinitely long". Also neglect temperature variation in the r-direction.

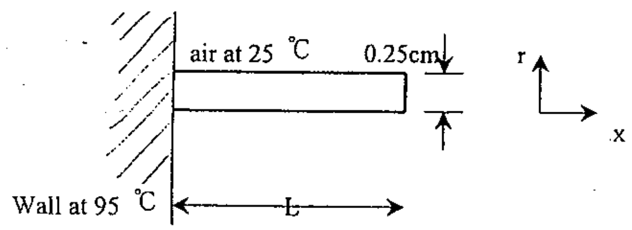
(a)Why can you assume T=T(x), (i.e. neglecting temperature variation in the r-direction.

(b)Derive the governing equation for T(x).

(c)Boundary conditions for the equation in (b).

(d)Solve for T(x).

(e)Calculate the heat loss through the fin.



(20%) 7. Consider the thermal problem of a vertical cylinder in a horizontal air flow. The convective heat transfer coefficient is h. The air has kinematics viscosity ν, thermal conductivity κ_f, thermal diffusivity α, density ρ, and volumetric coefficient of thermal expansion β. Gravitational acceleration is g. The thermal conductivity of the cylinder is κ_s. Other physical quantities are marked in the figure. Define the following dimensionless parameters for the problem in terms of the symbols appear in the question, and give physical significance for each parameter

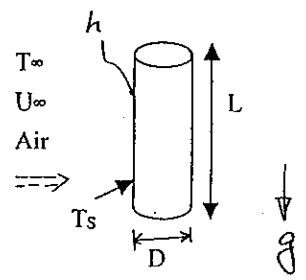
(a) Prandtl number

(b) Reynolds number

(c) Biot number

(d) Nusselt number

(e) Grashof number



科目：固體力學【機電系乙組】

Prob. #1 (40%)

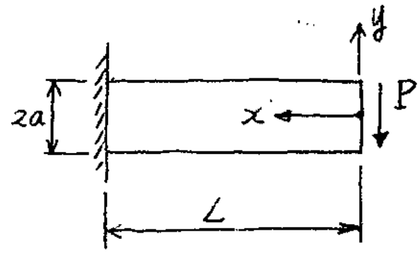
The stress function for a cantilever beam loaded by a shear force P at the free end is

$$F = C_1 x y^3 + C_2 x y$$

(a) Evaluate the constant C_1 and C_2. (10%)

(b) Derive the expressions for the displacements u and v. (20%)

(c) Compare v with the expression derived for displacement y from elementary bear theory, $EI(d^2y/dx^2) = M$. (10%)



Prob. #2 (30%)

The stress tensor has components

$$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

in a given system of units. Find the stress vector on an element of surface with normal in the direction of the vector (0, 1, 1). Give the normal and shear components of this stress vector. Also find the principal stresses and the direction cosines of the principal axes.

Prob. #3 (30%)

If a_{ij} is a second order anti-symmetric tensor, the associated (axial) vector is

$$b_i = -\frac{1}{2} e_{ijk} a_{jk}$$

Show that $\vec{b} = (-a_{23}, -a_{31}, -a_{12})$ and that $a_{rs} = -e_{irs} b_i$.

國立中山大學九十二學年度博士班招生考試試題

科目：自動控制【機電系丙組】

單選選擇題共 20 題，每題答對得 5 分，共 100 分，不倒扣。

(1). When we sketch the root locus on s-plane for a unity-feedback system with $G(s)=K(s^2-4s+20)/[(s+2)(s+4)]$, the angle of arrival for the complex zero, $s=2+j4$, is about (A) $-158°$ (B) $-58°$ (C) $38°$ (D) $168°$ (E) the above all wrong.

(2). When we sketch the root locus for a unity-feedback system with $G(s)=K/[s(s+1)(s^2+4s+13)]$, the root locus branches that extend to the right-half s-plane cross the imaginary axis at $s=\pm j\omega$, and ω is about (A) 0.81 (B) 1.92 (C) 2.43 (D) 3.24 (E) the above all wrong.

(3). We use MATLAB M-file to sketch the root locus for a unity-feedback system with $G(s)=K/[s(s+1)(s^2+4s+13)]$, and the root locus branches that extend to the right-half s-plane cross the imaginary axis at $s=\pm j\omega$. What statement we use to find out $j\omega$, and gain K at this $j\omega$? (A) [poles, K] = rlocfind(num,den) (B) [K, poles] = rlocusfind(num,den) (C) [poles, K] = rootfind(num,den) (D) [K, poles] = locusfind(num,den) (E) the above all wrong.

(4). We use MATLAB M-file to sketch the root locus for a unity-feedback system with $G(s)=K(s+2)/(s^2+2s+3)$, and we need to find out the intersection of a requested damping ratio ζ with the root locus. What statement we can employ? (A) sgrid(natural_frequency, damping_ratio) (B) grid(natural_frequency,damping_ratio) (C) sgrid(damping_ratio,natural_frequency) (D) grid(damping_ratio, natural_frequency) (E) the above all wrong.

(5). If there are Z zeros, and P poles in the right half s-plane for $1+GH(s) = 0$ (characteristic equation), and N is the number of clockwise encirclement of $-1+j0$ point of the conformal mapping GH(s) in GH(s)-plane, then for a stable closed-loop system, we get (A) N=P (B) N=−P (C) Z=−N (D) Z=N (E) the above all wrong.

(6). If $G(s)=K/[s(s+1)(s+5)]$, for a unity-feedback closed-loop system, then for K=10, the gain margin is close to (A) 10dB (B) −10dB (C) 20dB (D) −20dB (E) the above all wrong.

(7). If a unity-feedback closed-loop system with $G(s)=K/[s(s+1)(s+5)]$ is given, then for K=100, the phase margin is close to (A) $-40°$ (B) $40°$ (C) $-24°$ (D) $24°$ (E) the above all wrong.

(8). A closed-loop system with the characteristic equation, $s(s+4)(s^2+2s+2)+K(s+1) = 0$, then the asymptotes centroid, σ_A, is about (A) −0.51 (B) −1.24 (C) −1.67 (D) −1.96 (E) the above all wrong.

(9). Which of the following statement is right? (A) the bandwidth of $1/(s+1)$ is larger than $1/(3s+1)$ (B) the response of $1/(3s+1)$ is faster than $1/(s+1)$ (C) the time constant of $1/(s+1)$ is larger than $1/(3s+1)$ (D) the damping ratio of $1/(3s+1)$ is larger than $1/(s+1)$ (E) the above all wrong.

(10). For the sinusoidal transfer function of the second order system $\omega_n^2/(s^2+2\zeta\omega_n s+\omega_n^2)$ (ζ is damping ratio), the resonant peak, $M_{p\omega}$, is taken place at frequency (A) $\omega=\omega_n(1-\zeta^2)^{1/2}$ (B) $\omega=\omega_n(1-\zeta^{1/2})$ (C) $\omega=\omega_n$ (D) $\omega=\omega_n(1-2\zeta^2)^{1/2}$ (E) the above all wrong.

(11). For $T(s)=10/(s^5+2s^4+3s^3+6s^2+5s+3)$, the right half s-plane poles that the transfer function exists are (A) 1 (B) 2 (C) 3 (D) 4 (E) the above all wrong.

(12). Which of the following statement is right? (A)nonminimum-phase system is unstable (B) minimum-phase system has no left half s-plane poles.(C) nonminimum-phase system owns right half s-plane zeros(D) minimum-phase system owns no right half s-plane zeros(E) the above all wrong.

(13). For the Nyquist plot of $G(j\omega)=K/[j\omega(j\omega T+1)]$, $H(j\omega)=1$, as $\omega\to\infty^+$, $|GH(j\omega)|=0$, and $\angle GH(j\omega)$ = (A) $-270°$(B) $-180°$(C) $-90°$(D)$0°$(E) the above all wrong.

(14). A closed-loop system with an open-loop transfer function, $GH(s)=K/[s(s+3)(s^2+2s+2)]$, is given, then the angle of departure of the root locus for the pole $s=-1+j$ is about (A)$18°$(B)$108°$(C) $-72°$(D) $-112°$(E)the above all wrong.

(15). Given the open-loop transfer function $GH(s)=K/[s(T_1s+1)(T_2s+1)]$, and from the Nyquist plot and Nyquist stability criterion, (A)the closed-loop system is stable with arbitrary K(B)the closed-loop system is stable with $K>1$(C)the closed-loop system is unstable with $K>0$(D)the closed-loop system is unstable with K is real large(E) the above all wrong.

(16). Which of the following statement is right?(A)gain margin $K_g\equiv 1/|GH(j\omega_g)|$, where ω_g is the gain crossover frequency. (B)gain margin $K_g\equiv 1/|GH(j\omega_g)|$, where $\omega_g=-180°$(C) gain margin $K_g\equiv 1/|GH(j\omega_p)|$, where ω_p is the phase crossover frequency (D)gain margin $K_g\equiv 1/|GH(j\omega_p)|$, where $\omega_p=-180°$(E) the above all wrong.

(17). Which of the following statement for MATLAB M-file is right?(A) statement "margin(num,den)" can achieve phase margin data (B) statement "bode(num,den)" can achieve phase margin data (C)statement "bode(num,den)" can achieve Bode diagram in normal magnitude(D) statement "margin(num,den)" can achieve gain margin on Bode diagram in normal magnitude(E) the above all wrong.

(18). For the linear time-invariant system, $dx(t)/dt=Ax(t)+Bu(t)$, with a state-feedback control, $u(t)=-Kx(t)$ (if K is given), is applied, then we can use the Routh-Hurwitz stability criterion to test the stability of the closed-loop system with (A)the characteristic equation as $\det(sI-(A+BK))=0$(B)the characteristic equation as $\det(sI-(A-BK))=0$(C)the characteristic equation as $\det(sI+(A+BK))=0$(D)the characteristic equation as $\det(sI+(A-BK))=0$(E) the above all wrong.

(19). The steady-state velocity error constant $K_v=20$, is required for the design of a compensator $G_c(s)$ for a unity-feedback system with $G_cG(s)$ as the open-loop transfer function, and it means (A)input function is a unit-step function(B)the steady-state error is 0.05 due to unit-ramp input(C)the compensator $G_c(s)$ is a phase-lead compensator(D)the closed-loop system is unstable(E) the above all wrong.

(20). When we take a second-order "Pade approximation" for the delay term e^{-sT} (dalay time T=1) of a unity-feedback system with $G(s)=3e^{-sT}/(1+4s)$ as the open-loop transfer function, we use the MATLAB statement as [num,den]=pade(m,n), and (A)m=1,n=2 (B)m=2,n=1 (C)m=4,n=1 (D) m=3,n=2 (E) the above all wrong.

國立中山大學九十二學年度博士班招生考試試題

科目：**機械設計與製造【機電系丁組】** 共 / 頁 第 / 頁

一、試說明受到純彎曲(Pure bending)的樑(Beam)的截面常採用工字樑或C型樑的理由。(10%)

二、試說明機械元件受靜態負荷 (Static loading)時，我們常以失效理論 (Failure theory)為依據判定機械元件之安全性的原因。(10%)

三、(a) 何謂機械元件的持久限（Endurance limit）？(4%)

(b) 說明實驗室所得到的理論持久限 Se'與機械元件的實際持久限 Se 不同的原因，工程師應如何修正 Se 而得到機械元件的正確持久限。(6%)

四、說明鏈條傳動(Chain drives)的多邊形作用(Chordal action)及其產生原因。(10%)

五、(a) 說明漸開線齒型較擺線齒型常用於動力傳動系統的原因。(6%)

(b) 試簡述齒輪傳動常見的兩種失效形式。(4%)

六、寫出三種齒輪加工方法並簡述其原理。(20%)

七、說明共析鋼的正常化、淬火與回火等熱處理程序及金相組織變化。(20%)

八、寫出五種非傳統加工方法。(10%)