1. Find the general solution of the differential equation of $x^3y''' + 2x^2y'' + 4xy' - 4y = 3x^{-2}$ (x > 0). (15%)

2. Let $\delta(x,y,z)$ be the density at (x,y,z). Find the mass and the center of the cone $z = \sqrt{x^2+y^2}$, $0 \le x^2 + y^2 \le 4$ if $\delta(x,y,z) = x^2 + y^2$. (20%)

3. Compute the indicated power of the matrix; (10%)

$$A = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}; \qquad A^{31}.$$

4. Transform each of the following equations into Sturm-Liouville form, i. e. $[ry']' + (q + \lambda p)y = 0$.

(a) $xy'' + (1-x)y' + \lambda y = 0$ (Laguerre's equation). (10%)

(b) $y'' - xy' + \lambda y = 0$ (Hermite's equation). (10%)

5. Let α and β be distinct positive numbers. Show that (15%)

$$\int_{-\infty}^{\infty} \frac{1}{(x^2+\alpha^2)+(x^2+\beta^2)}dx = \frac{\pi}{\alpha\beta(\alpha+\beta)}.$$

6. Find the inverse Laplace transform of the functions. (20%)

(a) $\dfrac{1}{(s^2+\beta^2)^2}$; (b) $\dfrac{s^2}{(s^2+\beta^2)^2}$.

1. (8%)Write down the general Navier-Stokes equation. What conditions are required for the validity of the Navier-Stokes equation.

2. (7%)Write down the general Bernoulli equation. What conditions are required for the validity of the Bernoulli equation.

3. (30%)For laminar flow in a circular tube with constant fluid properties：
   a. What is a fully-developed flow ? (3%)
   b. What is a thermally fully-developed flow ? (3%)
   c. Prove that for a fully-developed flow, the Nusselt number (Nu) is constant. (6%)
   d. What are the values of Nu for fully-developed flows with constant wall temperature and constant wall heat flux, respectively? (4%)
   e. How to determine the thermal entrance length ? (4%)
   f. Draw a diagram to show how the heat transfer coefficient (h) varying along the flow direction (x) from the tube entrance to fully-developed. (5%)
   g. For the condition with constant wall heat flux, draw a diagram to show how the wall temperature (T_w) and the bulk mean temperature (T_b) varying along the flow direction (x) from the tube entrance to fully-developed. (5%)

4. (30%)For external boundary layer flow with constant fluid properties：
   a. What is a flow boundary layer? What is a thermal boundary layer? (3%)
   b. Write down the steady two-dimensional flow and thermal boundary layer equations. (4%)
   c. Properly define dimensionless variables and parameters and Non-dimensionalize both flow and thermal boundary layer equations. (5%)
   d. Define the friction coefficient and heat transfer coefficient for a boundary layer flow. Use dimensionless variables to indicate the friction coefficient and the Nusselt number. (4%)
   e. When a wall is approximately flat, what will the dimensionless boundary layer equations in c become? What dimensionless parameter determines which boundary layer (flow or thermal) develops faster? What is the physical meaning of that parameter? (5%)

f. What is the Reynolds analogy? Write down the conditions for the Reynolds analogy. (4%)

g. What is the modified Reynolds (or Chilton-Colburn) analogy? Write down the conditions for laminar flow that the modified Reynolds analogy can be applied. Also write down the conditions for turbulent flow that the modified Reynolds analogy can be applied. (5%)

5. (15%)For heated vertical flat plate free convection problem：

a. Write down the governing equations.(3%)

b. What is the Boussinesq approximation (3%)

c. Properly define dimensionless variables and parameters and Non-dimensionalize the governing equations.(5%)

d. Define the Grashof number (Gr). What is the physical meaning of Gr? Define the Rayleigh number (Ra).(4%)

6. (5%)How to determine if the flow is laminar or turbulent for internal flow, external flow and natural convection flow, respectively.

7. (5%)How to determine if a convection problem is forced convection dominant, free convection dominant or mixed.

1. Using appropriate sketches or words define the most general definitions of following terminologies: (50%)

(a) Plane stress and plane strain

(b) Bernouli-Euler Beam and Timnosinko Beam

(c) Stresss intensity factors K_I, K_{II} *and* K_{III}

(d) Hertz Contact Stresses

(e) Engineering stresses, Engineering strain, True stress and True strain

(f) Creep Deformation

(g) Strain Hardening

(h) Buckling

(i) Principal normal stresses, Principal shear stresses

(j) Residual stresses, Thermal stresses

2. (a) State in words and mathematically, the definition of failure according to

(i) the maximum normal stress theory (10%)

(ii) the maximum shear stress theory (10%)

(iii) the maximum distortion energy theory (10%)

(b) Consider the stress tensor at a specified point is

$$\left[\sigma_{ij}\right]=\begin{bmatrix}\sigma_{xx} & \tau_{xy} & \tau_{xz}\\ \tau_{xy} & \sigma_{yy} & \tau_{yz}\\ \tau_{xz} & \tau_{yz} & \sigma_{zz}\end{bmatrix}=\begin{bmatrix}31.83 & 31.83 & 0\\ 31.83 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} kpsi$$

Material under consideration for this problem is AISI steel. The ultimate tensile strength is $S_u =$ 60 kpsi and the yield strength is $S_y =$43 kpsi.

(i) To find the principal normal stresses $\sigma_1, \sigma_2, \sigma_3$ and the maximum principal shear stress τ_{max} at this point. (5%)

(ii) Is it possible to have a brittle failure by employing the maximum normal stress theory of failure ? (5%)

(iii) Is it possible to have a yielding failure by employing the maximum shear stress theory of failure ? (5%)

(iv) Is it possible to have a yielding failure by employing the maximum distortion energy theory of failure ? (5%)

(Note! The vonMises stress is defined as

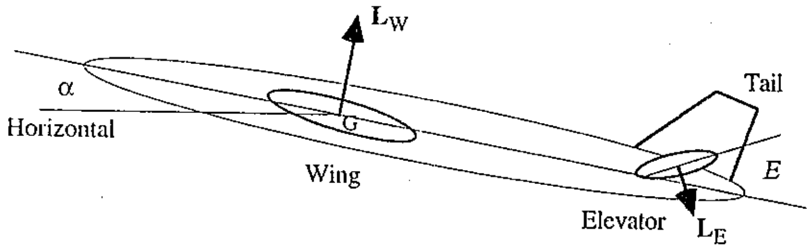
$$\sigma_f = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}}$$
)

國立中山大學八十九學年度碩博士班招生考試試題

科　目：機械所丙組（自動控制）　　　　共一頁　第一頁

<u>Altitude control of an Aircraft</u>

The sum of the lift forces applied to aircraft wings is equivalent to a single lift force L_W, applied at the 'center of lift'. For simplicity, we assume that the center of lift coincides with the center of mass G of the aircraft. The aircraft is initially at a constant altitude $h = h_0$. To affect its vertical motion, the elevator (a small surface located at the aircraft tail) is rotated by an angle E. This generates a small aerodynamic force L_E on the elevator, and thus a torque about G. This torque creates a rotation of the aircraft about G, measured by an angle α. The lift force L_W applied to the wings is proportional to α. Similarly, L_E is proportional to the angle between the horizontal and the elevator. Further, various aerodynamic forces create friction torques proportional to $\dot{\alpha}$



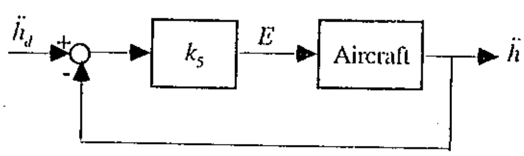
(1) Assuming that all angles are small enough to justify linear approximations, show that a simplified model of the aircraft vertical motion is:

$$\ddot{h} = k_1\alpha - k_2(E - \alpha)$$
$$\ddot{\alpha} = k_3(E - \alpha) - k_4\dot{\alpha}$$

where k_1, k_2, k_3 and k_4 are positive constants. (20%)

(2) Assume that $k_1 = 2$ and $k_2 = k_3 = k_4 = 1$. Draw the responses in altitude h and angle α to a unit step input at E. What are the initial and final vertical accelerations of h and α? (40%)

(3) Continued from (2), consider the control system



Determine the range of gain k_5 for assuring the system's stability. (20%)

(4) Continued from (2), investigate if it is possible to design an altitude controller for the aircraft.

- First try using only h as a measurement (10%)
- Then try applying both h and $\dot{h}$ (10%)

國立中山大學八十九學年度碩博士班招生考試試題

科　目：機械所丁組（機械設計與製造）

1. 試詳述在機械設計中所遇到之各種公差與配合及其在製造上之意義。 20%

2. 何謂 CAD, CAE, CAM, CIM, FMS？ 各舉例說明之。 20%

3. 試申論機械設計與機械製造之相互關係。 20%

4. 在 IC 相關產業日新月異之時代，你認為機械設計與製造領域，應擔任何種職責，試詳述之。 20%

5. 假設你就讀博士班時需要自製一套實驗設備，請詳述你將如何規劃、設計、製造及完成。 20%